SP-III/Mathematics/304SEC-1(T)/19

B.Sc. Semester III (General) Examination, 2018-19

MATHEMATICS

Course ID : 32110 **Course Code : SPMTH-304SEC-1(T)**

Course Title : Logic and Sets

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* questions: 5×2=10

(a) If $A = \{3, 4, 5, 7, 9\}, B = \{5, 9, 1, 6\}$ and $C = \{3, 2\}$, find all elements of the set $(A\Delta B) \times C$, when $\Delta=$ symmetric difference of two sets.

(b) List the elements of $A = \{x : x \in \mathbb{N}, 4 + x = 3\}$. Where $\mathbb{N}$ is the set of natural number.

(c) If $A = \{2, 12, 32\}$ and $B = \{1, 4, 8, 16\}$ and the universal set $\cup = \{1, 2, 4, 8, 12, 16, 32\}$ then prove that $(A \cap B)' = A' \cup B'$ where A' represents the complement of *A*.

(d) If $n(A) = 20,\ n(B) = 35$ and $n(A \cup B) = 45$ by drawing Venn-Euler diagram. Show that $n(A \cap B) = 10$.

(e) Write down the elements of the power set of the set $X = \{x, y, z, w\}$.

(f) Find the truth table of the conjunction of two statements *p* and *q*.

(g) Find the truth table of the disjunction of two statements *p* and *q*.

(h) If $A = \{1, 2\}, B = \{2, 3\}, C = \{3, 4\}$, find $A \times (B \cup C)$.

2. Answer *any four* questions: 5×4=20

(a) If A, B, C are subsets of the universal set *X*, prove the following:

(i) $(A \cap B) \cup (A \cap B') \cup (A' \cap B) \cup (A' \cap B') = X$

(ii) $(A \cup B \cup C') \cap (A \cup B' \cup C') = A \cup C'$. 3+2=5

(b) If A, B, C are subsets of an universal set S. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(c) If $n(A) = 100, n(B) = 90, n(C) = 100, n(A \cap B) = 60, n(B \cap C) = 40, n(A \cap C) = 45,$ $n(A \cup B \cup C) = 200$, find $n(A \cap B \cap C)$.

(d) Let p, q, r be statements. Then show that distributive law,

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$ holds by truth table.

(e) Find whether the relations R_1 and R_2 as defined below in the set $A = \{1, 2, 3\}$ are

(i) reflexive, (ii) symmetric, (iii) transitive.

(a) $R_1 = \{(2,1), (1,2), (3,3)\}$

(b) $R_2 = \{(3,3)\}$ 3+2=5

(f) If R be a relation in the set of integers $\mathbb{Z}$ defined by $R = \{(x, y) : x \in \mathbb{Z}, y \in \mathbb{Z}, (x - y)$ is divisible by $6\}$ then prove that R is equivalence relation. Find all the distinct equivalence classes of the relation R.

3. Answer *any one* question: 10×1=10

(a) (i) Let $\mathbb{Z}$ be the set of all integers and A, B, C, D are the subsets of $\mathbb{Z}$ given by,

$A = \{x \in \mathbb{Z} : 0 \leq x \leq 10\}$, $B = \{x \in \mathbb{Z} : 5 \leq x \leq 15\}$

$C = \{x \in \mathbb{Z} : x \geq 5\}$, $D = \{x \in \mathbb{Z} : x \leq 15\}$ then find $A \cup B, A \cap B, B - C, A - D$.

(ii) Let p, q and r be statements. Then show that De Morgan's law

$\sim(p \vee q) = (\sim p) \wedge (\sim q)$ holds by truth table. (2+2+1+1)+4=10

(b) (i) If R be an equivalence relation on the set A, then show that R^{-1} is also an equivalence relation on *A*.

(ii) If $a \equiv b\ (mod\ m) and\ C \equiv d(mod\ m)$ then prove that $a + c \equiv (b + d)(mod\ m)$ and $ac \equiv bd(mod\ m)$. (2+2+2)+(2+2)=10
